Poster Session

Adaptive models for Envelope Generation from 3D Video

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This poster introduces preliminary results in the dynamic modeling of actors in motion from irregularly distributed sparse data. This data has been captured from synchronized color and depth cameras (RGB-D). Once processed and analyzed, this data provides geometric and radiometric primitives to be managed in 3D video. The first step involves the resolution of technological issues concerning synchronization, signal conversion, process parallelization, or information fusion arising from different sensors, among others.

Modeling of human motion is a highly complex problem, augmented in those scenes with more than one actor. From the early years of the 21st century, kinematic modeling of mobile actors have been accomplished with different 3D video techniques applied to multimedia sectors. In our case, 3D video will be used to generate 3D representations of evolving actors in a structured scene from a finite collection of synchronized fixed cameras. Most models are centered in adjusting PL-structures or PS-structures to evolving clouds of points. This publication intends to provide some insights for the required algebraic models.

From a more theoretical viewpoint, there are a lot of unsolved mathematical problems related to 3D+1d modeling of actors. The inputs are related with the discrete nature of the point clouds with variable density depending on the relative orientation of the cameras, the illumination conditions, the clothes of the actors and the characteristics of the performance. In order to simplify, we will constraint ourselves to a scene with two actors performing slow motions with basic garment, avoiding finer details as those related with gestures or wrinkles.

The most relevant aspects concerning the *static analysis* (geometric initialization) include the following tasks: Clustering points following geometric and radiometric constraints. Automatic generation of local quadrangular meshes for well-defined clusters beyond a threshold. Local adjustment of low-degree surfaces for each quadrangular mesh. Overlapping of common regions (observed from different localizations of cameras) by using projective transformations. Generation of local algebraic models by using generalized B-spline surfaces of bidegree (d_1, d_2). Propagation of well-defined low-degree pieces by using parabolic flows to fill out unnatural holes. Matching by imposing low order contact conditions in the space of deformations. Optimization by using curvature flows for matched static pieces. Regularity constraints give a Zariski open set of generalized splines as ambient space for the above constructions. However, the presence of "events" (supported on Discriminant Loci for each projection map), motivates the need of considering low corank and low codimension singularities for finer moduli spaces. At the current state, our research only considers local models with singularities linked to generic projections of surfaces.

Motion is geometrically translated in evolving algebraic surfaces (a slice for each time step). This leads to more complicated singularities in a space-time representation. A coarse-to-fine approach allows to manage variable information. This approach stars with generic properties of surface fibrations (Hurewicz) but then it studies the "descent" to more restricted algebraic surfaces giving a fibration of an algebraic threefold with ordinary singularities (stable by deformations).

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Robust Mathematical Modeling for Motion from Video Sequences with a hand-held camera

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Visual odometry plays an important role in Simultaneous Localization and Mapping. The main goal is to compute the camera pose using only passive sensor. More specifically, monocular tracking is focused only on sequences captured with a single lens camera. The sparse approach to the problem consists in abstracting the image in a finite collection of features matched in real-time (around 25 fps). The hand-held camera moves along a static environment (indoor or traffic scenes, e.g.) to achieve autonomous navigation.

There exist partial solutions to the problem which have been already applied in automatic navigation. However, to achieve a more robust (in regard to the scene), and adaptive (in regard to possible events) solution, it is necessary to solve some mathematical issues, such as: (1) detection, tracking and clustering of trajectories linked to salient mobile features with geometric (corners, e.g.) or radiometric information (light intensity, e.g.); (2) estimation of kinematics with a feedback between real 3D+1d scene flow Φ and apparent 2D+1d image flow $\phi = \pi(\Phi)$; (3) prediction of different events which are modeled as singularities of (scalar, vector, tensor) fields.

Our approach to solve the problem is based on tracking and clustering these features in adaptive packs of trajectories. The development of a structural kinematic model allows to integrate different probabilistic, geometric, algebraic, and topological traits in a common framework. More especifically, and by following a coarse-to-fine approach, we have considered:

Probabilistic traits, which approximate localization of point clouds with similar kinematic behavior. Local trajectories fulfilling "contact constraints" are used, generating coarse envelopes after outliers removal, and evaluating statistical measures of global (energy vs entropy) functionals.

Geometric traits, which coarse reconstructions generated by a "weighted" combination of homographies (relative to perspective models in structured scenes) and fundamental matrices (for non-structured scenes). The introduction of total variation methods (reminiscent of the total energy functional) provides a key to evaluate the "proximity" between partially reconstructed models and a first validation. To achieve an intrinsic presentation it is necessary to incorporate actions groups linked to the feedback between rigid structure of the scene (Euclidean Group) and evolving appearances (Special Linear Group).

Algebraic traits, which provide invariant representation (linked to the coadjoint action) for some meaningful classical groups. This representation is based on a feedforward strategy between $SL(n) \ltimes \mathbb{R}^n$ (for apparent deformations in the image and scene flow up to scale) and $E(n) := SO(n) \ltimes \mathbb{R}^n$ (for robust euclidean information). The transference of information is performed on the common universal covering for their complex version.

Topological traits, which model "small deformations" for regular variation of packages of trajectories (invariants of the homotopy class of maps between successive images). Alternately, "sudden events" can be modeled in terms of singularities of (scalar, vector, tensor) fields.

Coauthors: Javier Finat Codes, Belén Palop del Río.

Moduli of Hypersurfaces in P^3

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We define a compactification of the moduli space of degree d hypersurfaces H in P^3 using pairs (X, D) corresponding to degenerations of (P^3, H). We show that strictly slc Fano varieties have a certain form and use that classification to show that the moduli space is a proper Deligne-Mumford stack for odd degree d and that the pairs appearing have at worst slt singularities. Most of this work relies on the study of singularities and the minimal model program.

Humbert-Edge curves

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We discuss W. L. Edge's approach to Humbert's curves as a canonical genus 5 curve that is a complete intersection of three diagonal quadrics. Moreover, the contributions of Edge to the study of projective curves in $\mathbb{P}^n$ that are complete intersection of $n-1$ quadrics are explained and we present some results complementary to the Edge's exposition.

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Local information of difference equations

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The theory of algebraic differential equations on the affine line is very well-understood. In particular, there is a well-defined notion of restricting a differential equation (which we think of as a D-module) to a formal neighborhood of a point, and D-modules over a formal neighborhood over a point are completely described by two vector spaces, called vanishing cycles and nearby cycles, and some maps between them. We give an analogous notion of "restriction to a formal disk" for difference equations that satisfies several desirable properties: first of all, a difference module can be recovered uniquely from its restriction to the complement of a point and its restriction to a formal disk around this point. Secondly, it gives rise to a notion of vanishing cycles that is compatible with the Mellin transform, in that vanishing cycles of a difference module are determined by nearby cycles of its Mellin transform, which is a D-module on the torus. This relation is the local Mellin transform, which is analogous to the local Fourier transform for D-modules.

Vertex algebras and Hodge estructures

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In this short note we discuss some natural inter-relations between Hodge structures and vertex algebras of conformal field theory. Some part of this on a correspondence between Higgs bundles and opers already is known in the literature as non-abelian Hodge theorem due to C. Simpson. The same kind of correspondence has been well studied over flag manifolds of semisimple Lie groups known as Beilinson-Bernstein localization. Our goal is to explain how the data of a variation of Hodge structure as a solution of a regular holonomic system is matched with similar system of vertex algebra modules arising in conformal field theory. The result of the discussion is an analogue of the Bernstein correspondence over a local manifold. We associate to flat connections of mixed Hodge structures a generalized version of Harish-Chandra modules called Wakimoto modules and a generalized Harish-Chandra homomorphism. Therefore the map of correspondence is a more developed form of Harish-Chandra isomorphism. This text mainly proposes to motivates some ideas of representations of vertex algebras into Hodge theory. We have brought the basic ideas in the two fields close to each other. We enclose with an explanation of geometric Langlands correspondence as a generalization of the discussion.

On volumes of complements of periodic geodesics

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Every closed geodesic γ on a surface has a canonically associated knot $\hat{\gamma}$ in the projective unit tangent bundle. We study, for γ filling, the volume of the associated knot complement with respect to its unique complete hyperbolic metric.

Adler-Moser potentials and Differential Galois Theory

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We consider Schrödinger equation

$$(L-E)\phi = -\phi_{xx} + (u-E)\phi = 0$$

with Adler-Moser potentials defined in [AM]. We compute explicit fundamental matrices of this equation for two particular Adler-Moser potentials and show how these fundamental matrices are related performing Darboux transformations. Finally, applying Thm 2.2 of [JMSZ], we show that their differential Galois groups are the same and time-independent.

This argument can be generalized for all Adler-Moser potentials, and hence we can compute their differential Galois groups.

This is a joint work with Sonia Jiménez, Juan J. Morales Ruiz and María-Ángeles Zurro.

References

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