Plenary lectures

Lefschetz structure on Vanishing Cohomology

Xavier Gómez-Mont

CIMAT, Mexico

Thursday 25, 18:00-19:00

The Hard Lefschetz Theorem and the Riemann-Hodge bilinear relations are deep theorems that structure the cohomology groups of projective varieties. Relative versions of these theorems can be formulated for smooth families of projective varieties, say over a disc, giving structure to the cohomology bundles, that have the flat Gauss-Manin connection . When we have a family of varieties over the disc $f : X \to D$, which is smooth over the punctured disc, besides the above structure on the cohomology bundle of the punctured disc, we have the monodromy map. The Monodromy Theorem asserts that the eigenvalues of the monodromy map are roots of unity, so after a suitable base change of the disc branched at 0 we obtain a family of varieties whose monodromy map is unipotent, and its logarithm N is a well defined nilpotent map acting in cohomology. This map N carries the information of the non-semisimple part of the monodromy action. There is an interesting interplay between the filtration induced by N, the Lefschetz and Hodge decomposition of cohomology and the bilinear form induced by cup product giving the Hodge-Riemann bilinear relations. On the algebraic side, when the singularity is isolated, we have the Jacobian Algebra, Grothendieck non degenerate bilinear pairing in the Jacobian Algebra and multiplication by f in the Jacobian Algebra. This setting allows also to do Lefschetz type decomposition, as above. The relation between the topological picture and the algebraic picture is via a Theorem of Varchenko, that asserts that N is multiplication by f when we take the graded module with respect to the Mixed Hodge structure in the Jacobian Algebra. The objective of the presentation will be to describe the topological picture, the algebraic picture, and compare both. This is joint work with M.A. de la Rosa, from Catedras CONACYT-UJAT.

Let's get Rational

Rubén Hidalgo

Universidad de La Frontera, Chile

Tuesday 23, 18:00-19:00

Due to Milnor, the moduli space M_d, of rational maps of degree $d \geq 2$, is known to have the structure of a complex orbifold of dimension $2(d-1)$. The **branch locus** $\mathcal{B}_d \subset \mathrm{M}_d$ is given by the classes of rational maps with non-trivial group of holomorphic automorphisms. Miasnikov, Stout and Williams have recently observed that every finite group of $\mathrm{PSL}_2(\mathbb{C})$ can be seen as the full group of holomorphic automorphisms of a suitable rational map and a description of these maps is provided in terms of the corresponding finite group.
Milnor also noted, by using the symmetric forms in the multipliers of the fixed points, that M_2 can be identified with $\mathbb{C}^2$. In this model, Fujimora noted that $\mathcal{B}_2$ corresponds to the cubic curve

$$2x^3 + x^2y - x^2 - 4y^2 - 8xy + 12x + 12y - 36 = 0.$$

If $d \geq 3$, $\mathcal{B}_d$ corresponds exactly to the sublocus of M_d where it fails to be a topological manifold. We observe that the branch locus $\mathcal{B}_d$ is always connected The conjugation action on rational maps by the reflection $J(z) = \overline{z}$ provides of a **real structure** on M_d. The locus $\mathrm{M}_d^{\mathbb{R}}$ of real points of such a structure consists to those rational maps admitting antiholomorphic automorphisms. If a rational maps admits a reflection (antiholomorphic involution having fixed points) as an automorphism, then it is called **real**. A rational maps admitting antiholomorphic automorphisms, but none of them being a reflection, is called **pseudo-real**. The locus $\mathrm{M}_d(\mathbb{R}) \subset \mathrm{M}_d$ consisting of real rational maps is a connected real orbifold of real dimension $2(d-1)$. If we denote by $\mathcal{P}_d \subset \mathrm{M}_d$ the locus consisting of the pseudo-real rational maps, then $\mathrm{M}_d^{\mathbb{R}}$ is the disjoint union of $\mathrm{M}_d(\mathbb{R})$ and $\mathcal{P}_d$. Silverman noted that $\mathcal{P}_d = \emptyset$ if d is even and that, for $d \geq 3$ odd, $\mathcal{P}_d \neq \emptyset$. We show that (for $d \geq 3$ ood) the locus $\mathcal{P}_d$ is always disconnected and that $\mathrm{M}^{\mathbb{R}}$ is always connected. We also have provide a description of the rational maps admitting an antiholomorphic automorphism.
Given a rational map R, there is an associated field $\mathcal{M}_R$, called its **field of moduli** (this is an invariant under the action of the group $\mathrm{Gal}(\mathbb{C})$). By results due to Koizumi, $\mathcal{M}_R$ is the intersection of all the fields of definitions of R. Silverman observed that, for either d even or R equivalent to a polynomial, this field is a field of definition of R. In the case that R cannot be defined over its field of moduli (so $d \geq 3$ must be odd), we have seen that it can be defined over a suitable quadratic extension of it.
The field $\mathcal{M}_R$ is a subfield of $\mathbb{R}$ if and only if R is either real or pseudo-real. Also, R can be defined over the reals if and only if it is real. In particular, pseudo-real rational maps are examples of rational maps not definable over their field of moduli. Explicit examples of pesudo-real rational maps, with trivial group of holomorphic automorphisms were provided by Silverman. We observe that, for a pseudo-real rational map, its group of holomorphic automorphisms is either trivial or a cyclic group. For each $n \geq 2$, we present explicit examples of

pseudo-real rational maps with a cyclic group of order n as group of holomorphic automorphisms.

In the case of a real rational map, it is more difficult to check if it can or not be definable over its field of moduli. We present explicit examples of real rational maps which cannot be defined over the filed of moduli.

Acknowledgements: Partially supported by FONDECYT 1150003 and Anillo ACT 1415 PIA-CONICYT.

Analytic parameterizations of limit shapes

Richard Kenyon

Brown University, USA

Wednesday 24, 12:00-13:00

The "5 vertex model" is a certain probability measure on discrete interfaces in 2+1 dimensions. In the scaling limit (when mesh tends to zero) a random sample converges almost surely to a fixed nonrandom smooth surface called a "limit shape". These limit shapes satisfy a variational principle, minimizing an associated surface tension. We show that they can be parameterized by analytic functions. This is joint work with Jan de Gier and Sam Watson.

Wow, so many minimal surfaces!

André Arroja Neves

University of Chicago, USA

Friday 26, 12:00-13:00

Minimal surfaces are ubiquitous in Geometry but their existence theory is rather mysterious. For instance, Yau in 1982 conjectured that any 3-manifold admits infinitely many closed minimal surfaces but the best one knows is the existence of at least two.

In a different direction, Gromov conjectured a Weyl Law for the volume spectrum that was proven last year by Liokumovich, Marques, and myself.

I talk about recent my work with Irie and Marques: we combined Gromov's Weyl Law with the Min-max theory Marques and I have been developing over the last years to prove that, for generic metrics, not only there are infinitely many minimal hypersurfaces but they are also dense.

A parametrization of the moduli space of abelian 6-folds

Angela Ortega

Humboldt Universität, Germany

Thursday 25, 12:00-13:00

It has been known for over a century that the general abelian variety of dimension at most five is either a Jacobian, or a Prym variety. This allows one to reduce the study of abelian varieties of small dimension to the rich and concrete theory of curves.
In this talk we will discuss recent decisive progress on finding a structure theorem for abelian varieties of dimension six, as Prym-Tyurin varieties associated to coverings of curves with E_6-monodromy, and the implications this uniformization result has on the geometry of the moduli space $\mathcal{A}_6$.
This is a joint work with V. Alexeev, R. Donagi, G. Farkas ans E. Izadi.

Tropical motivic integration

Sam Payne

Yale University, USA

Monday 22, 12:00-13:00

I will present a new tool for the calculation of motivic invariants appearing in Donaldson-Thomas theory, such as the motivic Milnor fiber, starting from a theory of volumes of semi-algebraic sets introduced a decade ago by Hrushovski and Kazhdan. They key new result for applications is a tropical Fubini theorem-the invariants of interest can be computed by integrating the volumes of fibers of the tropicalization map with respect to Euler characteristic on the base.

Classification of spheres with constant mean curvature in homogeneous three-manifolds

Joaquín Pérez

Universidad de Granada, Spain

Wednesday 24, 10:30-11:30

Surfaces with constant mean curvature arise as solutions of the variational problem associated to the area functional with volume constraint (isoperimetric problem). Two central results about these surfaces are the classical theorems by Hopf and Alexandrov, that characterize round spheres in $\mathbb{R}^3$ among immersions of spheres and among compact embeddings with constant mean curvature, respectively. Abresch and Rosenberg generalized Hopf theorem to the case of ambient Thurston geometries, which are simply connected homogeneous three-manifolds with isometry group of dimension four. In this talk we will explain how to extend these results to any Riemannian homogeneous three-manifold.

This is joint work with William H. Meeks, Pablo Mira and Antonio Ros.

Higgs bundles, branes and applications

Laura Schaposnik

University of Illinois at Chicago

Tuesday 23, 12:00-13:00

Higgs bundles are pairs of holomorphic vector bundles and holomorphic 1-forms taking values in the endomorphisms of the bundle, and their moduli spaces carry a natural Hyperkahler structure, through which one can study Lagrangian subspaces (A-branes) or holomorphic subspaces (B-branes). Notably, these A and B-branes have gained significant attention in string theory. We shall begin the talk by first introducing Higgs bundles for complex Lie groups and the associated Hitchin fibration through which one can realize Langlands duality. We shall then look at natural constructions of families of subspaces which give different types of branes, and relate these spaces to the study of 3-manifolds, surface group representations and mirror symmetry.

Equidistribution of square-tiled surfaces, meanders, and Masur-Veech volumes

Anton Zorich

Institut Mathématiques de Jussieu, Paris, France

Monday 22, 18:00-19:00

We show how recent results of the authors on equidistribution of square-tiled surfaces of given combinatorial type allow to compute approximate values of Masur-Veech volumes of the strata in the moduli spaces of Abelian and quadratic differentials by Monte Carlo method.

We also show how similar approach allows to count asymptotical number of meanders of fixed combinatorial type in various settings in all genera. Our formulae are particularly efficient for classical meanders in genus zero.

We construct a bridge between flat and hyperbolic worlds giving a formula for the Masur-Veech volume of the moduli space of quadratic differentials in terms of intersection numbers (in the spirit of Mirzakhani's formula for Weil-Peterson volume of the moduli space of pointed curves).

Joint work with V. Delecroix, E. Goujard, P. Zograf.