Special Session 6: Geometry and Physics

Organizers: Luis Miguel Nieto Calzada (Universidad de Valladolid, Spain), Miguel Sánchez Caja (Universidad de Granada, Spain) and Carlos Tejero Prieto (Universidad de Salamanca, Spain).

Kähler metrics from Lorentzian geometry in dimension four

Amir Babak Aazami

Clark University, Massachussets, USA

Thursday 25, 15:25-16:00

Given a Lorentzian 4-manifold (M, g) with two distinguished vector fields satisfying properties determined by their shear, twist and various Lie bracket relations, a family of Kähler metrics g' is constructed, defined on an open set in M, which coincides with M in many typical examples. Under certain conditions g and g' share various properties, such as a Killing vector field or a vector field with geodesic flow. Many examples are described, including de Sitter spacetime, gravitational plane waves, and Kerr spacetime.
This is joint work with Gideon Maschler.

Kähler-Yang-Mills equations and gravitating vortices

Luis Álvarez-Cónsul

ICMAT, Madrid, Spain

Thursday 25, 16:50-17:25

In this talk, we first introduce the Kähler-Yang-Mills equations for a compact manifold of Kähler type, equipped with a holomorphic bundle. They emerge from a natural extension of the theories for constant scalar curvature Kähler metrics and Hermitian-Yang-Mills connections. We construct solutions to these equations by applying dimensional reduction methods to the product of the complex projective line with a compact Riemann surface. The resulting equations on the Riemann surface, that we call gravitating vortex equations, describe abelian vortices coupled to a metric on the surface. In genus zero, they

include as a particular case the Einstein-Bogomol'nyi equations, that physically correspond to Nielsen-Olesen cosmic strings in the Bogomol'nyi phase. Based on joint work with M. García-Fernández, O. García-Prada and V. P. Pingali.

Hermite-Gauss functions and Fourier series

Enrico Celeghini

Istituto Nazionale di Fisica Nucleare, Italy and Universidad de Valladolid, Spain

Friday 26, 14:45-15:20

Hermite-Gauss functions play a fundamental role in the theory of Fourier Transform as they are eigenvectors of Fourier Transform, a basis of the L^2 functions on $\mathbb{R}$ and a representation of the Lie algebra $h(1)$.
We exhibit that, with an appropriate discretization and periodization, Hermite-Gauss functions cover the same role for the Fourier Series and its inverse, the Discrete Time Fourier Transform. Discretized Hermite-Gauss functions are indeed a basis in $\mathbb{Z}$, while periodized Hermite-Gauss functions are a basis for periodic functions in $\mathbb{R}$. By inspection, the two sets are related by Fourier Transform and its inverse. Moreover, the position operator X-consistently defined as the operator conjugate of the derivative- allows the construct again a representation of the algebra $h(1)$ in both spaces of discrete and periodic functions.

On a fully nonlinear version of the Min-Oo Conjecture

José M. Espinar

IMPA, Brazil

Friday 26, 08:45-09:20

In this talk, we prove that the Min-Oo's conjecture holds if we consider a compact connected locally conformally flat manifold with boundary such that the eigenvalues of the Schouten tensor satisfy a fully nonlinear elliptic inequality, and the mean curvature of the boundary is controlled bellow by the mean curvature of a geodesic ball in the standard unit-sphere. This is a joint work with E. Barbosa and M.P. Cavalcante.

The Schwinger Mechanism in Kaluza-Klein Manifolds

Guglielmo Fucci

East Carolina University, North Carolina, USA

Thursday 25, 08:45-09:20

In this talk we will present some recent results regarding the rate of particle creation for scalar fields due to a uniform electric field in Kaluza-Klein manifolds.

In particular, we utilize spectral zeta function regularization techniques in order to compute the one-loop effective action of the system and its corresponding imaginary part. The analysis relies heavily on explicit formulas obtained by the author concerning the Hurwitz zeta function of imaginary second argument. We will show that the presence of the extra-dimensions and their specific geometric characteristics, influence the rate at which pairs of particles and anti-particles are generated. The results presented could lay the foundations of an indirect method for detecting extra-dimensions.

Canonical metrics on (string) holomorphic Courant algebroids

Mario García Fernández

ICMAT, Madrid, Spain

Thursday 25, 16:10-16:45

Yau's solution of the Calabi Conjecture implies, in particular, that every projective Calabi-Yau manifold X admits a metric with holonomy contained in $SU(n)$, and that these metrics are parametrized by the Kähler cone on $H^2(X, R)$. In this talk I will give evidence of an extension of Yau's theorem to non-Kähler manifolds, where the Calabi-Yau manifold is replaced by a pair (X, Q), given by a compact complex manifold with vanishing first Chern class and a (string) holomorphic Courant algebroid. Holomorphic Courant algebroids appear naturally in conformal field theories in physics and are intimately related to Hitchin's theory of generalized geometry. Joint work with Roberto Rubio, Carlos Shahbazi and Carl Tipler.

Stability of the Poincaré bundle for the stack of principal G-bundles

Tomás Luis Gómez

ICMAT, Madrid, Spain

Friday 26, 10:10-10:45

Let X be a smooth projective curve of genus at least two. Let G be an almost simple affine algebraic group. The universal principal G-bundle is called the Poincaré bundle. It is a principal G-bundle on the product of the curve and the moduli stack of principal G-bundles. We prove that it is stable with respect to any polarization (joint work with I. Biswas and N. Hoffmann).

Submersions with positive sectional curvature

David González Álvaro

University of Fribourg, Switzerland

Thursday 25, 09:25-10:00

In this talk we will review some classical results concerning manifolds with positive sectional curvature, and we will discuss some related open problems. In particular we will study potential obstructions to the existence of Riemannian submersions between positively curved manifolds, based on joint work with Luis Guijarro and Marco Radeschi.

Gluing formulas and the Casimir effect

Klaus Kirsten

Baylor University, Waco, Texas, USA

Thursday 25, 10:50-11:25

Let M_1 and M_2 be two Riemannian manifolds each of which have the boundary N. Consider the Laplacian on M_1 and M_2 augmented with Dirichlet boundary conditions on N. A natural question to ask is if there is any relation between spectral properties of the Laplacian on M_1, M_2, and the Laplacian on the manifold M (without boundary) obtained gluing together M_1 and M_2, namely $M = M_1 \cup_N M_2$. A partial answer is given by the Burghelea-Friedlander-Kappeler-gluing formula for zeta-determinants. This formula contains an (in general) unknown polynomial which is completely determined by some data on a collar neighborhood of the hypersurface N. In this talk I present results for the polynomial in terms of suitable geometric tensors on N. Choosing M_1, M_2 and M suitably, a gluing formula for Casimir energies results. This relates Casimir forces to the Dirichlet-to-Neumann map, providing a completely new perspective.

A Plateau problem at infinity for minimal surfaces in $\mathbb{H}^2 \times \mathbb{R}$

Magdalena Rodríguez

Universidad de Granada, Spain

Friday 26, 10:50-11:25

In this talk we will discuss some existence and non-existence results for properly embedded minimal annuli with prescribed boundary at infinity. This is a joint work with Leonor Ferrer, Francisco Martín and Rafe Mazzeo.

Timelike surfaces in Minkowski space with a canonical null direction

Gabriel Ruiz

UNAM, Mexico

Thursday 25, 14:45-15:20

Given a constant vector field Z in Minkowski space, a timelike surface is said to have a canonical null direction with respect to Z if the projection of Z on the tangent space of the surface gives a lightlike vector field. For example, in the three-dimensional Minkowski space: a surface has a canonical null direction if and only if it is minimal and flat. When the ambient has arbitrary dimension, if a surface has a canonical null direction and has parallel mean curvature vector then it is minimal. We give different ways for building these surfaces in the four-dimensional Minkowski space. We describe several properties in the non ruled general case in four-dimensional Minkowski space.

A tale of two geometric gravitation theories: From Newton-Cartan's to Einstein's?

Mariano Santander

Universidad de Valladolid, Spain

Friday 26, 15:25-16:00

As a matter of historical fact, Einstein's theory of gravitation (ETG) was a complete breakthrough in our understanding of gravitation, for if compared to its antecedent, the newtonian theory (NTG), the realm of ideas involved in both theories seems to be worlds apart. Starting with Cartan in the 1920s, there has been a gradually growing realization that Newton's theory can be reinterpreted also as a theory of curvature of the (newtonian) space-time, reformulation which is known as Newton-Cartan (NC) gravity.

In the first part of this talk I will briefly review the Newton-Cartan interpretation of the NTG, paying attention to the fact that there are different newtonian connections compatible with the 'degenerate' metric and cometric in the NC theory. This can be also reformulated in terms of the infinite-dimensional group of rigid motions, which provides the context to single out a connection which describes the inertial and gravitational effects which are actually present in any reference frame.

The second part deals with a contrafactual question, to be seen as an example of fictional history with a moral. The question is: Had Einstein 1908-1915 been not there, could a physicist working in the 1920s and knowleadgeable about the then recent contributions of Levi-Civita and Cartan, as well about the basic role of the action principle, have reformulated the ordinary newtonian theory of gravitation in a geometric mood? Or, in other words, could things have happened so that newtonian gravitation was reformulated as a geometric theory *before the creation of the ETG*?

Folk wisdom says, probably rightly, that had Einstein not built the TEG en 1908-1915, no one would have done *as he did* in the next 20 or 30 years. But the Newton-Cartan's to Einstein's fictional path provides a plausible way to go from the old Newton's theory to the new ETG theory in a manner slightly different from the one Einstein actually did. In the talk I will explore these questions.

Constant angle hypersurfaces in semi-riemannian space-forms

Didier A. Solís Gamboa

Universidad Autónoma de Yucatán, Mexico

Thursday 25, 10:10-10:45

One of the main fields of study in classical differential geometry consists of the analysis and classification of submanifolds that satisfy certain geometric constraints. These conditions are often related to both curvature and structures present in the ambient space. In this talk we present some classification results for hypersurfaces immersed in semi-riemannian space-forms having a constant angle with respect to a closed and conformal vector field.

A characterization of the grim reaper cylinder

Eddygledson Souza Gama

Universidade Federal do Ceará, Brazil

Friday 26, 09:25-10:00

In this lecture, we will talk about a characterization of the grim reaper cylinder as the only translating soliton by mean curvature flow in R^n which is C^1-asymptotic to two parallel half-hyperplanes outside a cylinder. In fact, a stronger result of characterization holds: the grim reaper cylinder and the hyperplane are the unique examples of translating solitons in R^n which are C^1-asymptotic to two half-hyperplanes outside a cylinder.