Special Session 4: Algebraic and Complex Geometry and Applications

Organizers: Francisco J. Plaza Martín (Universidad de Salamanca, Spain) and Emma Previato (Boston University, USA).

Geometry and Topology of character varieties

Carlos Florentino

Universidade de Lisboa, Portugal

Friday 26, 14:45-15:20

Character varieties are spaces of representations of finitely presented groups F into Lie groups G. In some cases, notably when F is the fundamental group of a surface, these spaces can be interpreted as moduli spaces of G-Higgs bundles over Riemann surfaces, recently studied in connection with the geometric Langlands program, and with mirror symmetry.

When G is a complex algebraic group, character varieties are algebraic and have interesting geometry and topology. There are also more refined invariants such as Deligne's mixed Hodge structures, which provide relevant arithmetic information, but are typically very difficult to compute.

In this seminar, we present explicit computations of the mixed Hodge-Deligne polynomials, and of so-called E-polynomials, of G-character varieties of free, and free abelian groups, when G is a group such as SL(n,C), (P)GL(n,C) or Sp(n,C). We also comment on interesting relations between the free case and some formulas by Mozgovoy-Reineke on counting quiver representations over finite fields.

This is joint work with A. Nozad, J. Silva and A. Zamora.

A new approach to rational singularities

Sándor Kovács

University of Washington, USA

Thursday 25, 16:10-16:45

I will discuss a new point of view on rational singularities. This consists of two essential elements: replacing the use of resolutions of singularities with Macaulayfication (which is known to exist under very mild assumptions) and a Kempf-type criterion for rational singularities using these Macaulayfications. This new view-point allows the study of rational singularities in situations where resolution of singularities is not known to exist and it agrees with the traditional approach when resolutions do exist. In particular, this leads to new results in both zero and positive characteristic.

On the Connectivity of Branch Loci of Moduli Spaces of Riemann Surfaces

Milagros Izquierdo

Linköping University, Sweden

Thursday 25, 15:25-16:00

The moduli spaces of Riemann and Klein surfaces can be considered as orbifolds where the singular locus is formed by Riemann surfaces with automorphisms, *the branch loci*.

In this talk we see the different behavior of the moduli spaces of Riemann surfaces, where with a few exceptions the branch loci is disconnected and consists of many connected components, and Klein surfaces whose branch loci are, with a few exceptions, connected.

We will also present an example of an infinite family of genera g where the branch locus of the completion of the moduli spaces of Riemann surfaces of genus g is still disconnected.

The results are joint work with Antonio F. Costa, Hugo Parlier and Ana Porto.

Post-quantum cryptography with polynomials

Ignacio Luengo

Universidad Complutense de Madrid, Spain

Friday 26, 15:25-16:00

Post-quantum cryptography is the public-key cryptography resistant to future quantum computers. In this talk we will talk about a post-quantum cryptosystem based on multivariate polynomial applications that we have developed (using ideas of Algebraic Geometry), patented and present it to the NIST contest to choose the future post-quantum cryptography standard.

Recent proofs of Weil's reciprocity law

Fernando Pablos Romo

Universidad de Salamanca, Spain

Thursday 25, 16:50-17:25

In 1938 A. Weil proved in [6] a reciprocity law for meromorphic functions of a Riemann surface. In fact, if X is a complex Riemann surface, A. Weil proved that

$$f((g)) = g((f))\,,$$

for all funcions $f, g \in \Sigma_X$, and being

$$(f) = \sum_{x \in X} v_x(f) \cdot x\,.$$

By generalizing to the multiplicative case the genial proof of J. Tate of the Residue Theorem -[5]-, in 1989 E. Arbarello, C. de Cocini and V.G Kac offered a proof of the Weil's reciprocity law from the finiteness of the cohomology of a complete algebraic curve over an algebraically closed field -[1]-. In this talk, we present several proofs of the Weil's reciprocity law obtained during the last 15 years in the Department of Mathematics of the University of Salamanca: a generalization of the Arbarello-De Concini-Kac proof for complete curves over perfect fields -[4]-, a global proof using Sato Grassmannians -[2]- and a proof as a particular case of a general reciprocity law on arbitrary vector spaces -[3]-.

References

[1] Arbarello, E.; de Concini, C.; Kac, V.G., *The Infinite Wedge Representation and the Reciprocity Law for Algebraic Curves*, Proc. of Symposia in Pure Mathematics, Volume **49,** Part I, A.M.S., (1989) 171-190.

[2] Muñoz Porras, J.M., Pablos Romo, F., *Generalized reciprocity laws*, Trans. Amer. Math. Soc. **360 (7)** (2008) 3473–3492.

[3] Pablos Romo, F., *A general reciprocity law for symbols on arbitrary vector spaces*, J. Alg. Appl., to appear in 2018.

[4] Pablos Romo, F., *On the Tame Symbol of an Algebraic Curve*, Comm. Algebra **30(9),** (2002) 4349–4368.

[5] Tate, J. T., *Residues of Differentials on Curves*, Ann. Scient. Éc. Norm. Sup., 4a série **1,** (1968) 149–159.

[6] Weil, A., *Généralisation des fonctions abéliennes*, J. Math. Pures et Appl. **17,** (1938) 47–87.

Hilbert scheme of points and foliations on $\mathbb{CP}^2$

Claudia Reynoso Alcántara

CIMAT, Guanajuato, Mexico

Friday 26, 16:10-16:45

The Hilbert scheme of points in the projective plane can be stratified through the Hilbert function. A stratum correponds to an open set of the space of foliations on $\mathbb{CP}^2$. We will describe this correspondence and we will use the geometry of the singular scheme to say something about the singular points and solutions of some special foliations. In particular we will give new examples of foliations without algebraic solutions.

Finite Spaces

Fernando Sancho de Salas

Universidad de Salamanca, Spain

Thursday 25, 14:45-15:20

We will show how finite ringed spaces naturally appear in several branches of mathematics, in general, and in algebraic geometric, in particular.

Derived equivalences of Abelian varieties and symplectic isomorphism

Carlos Tejero Prieto

Universidad de Salamanca, Spain

Friday 26, 16:50-17:25

We study derived equivalences of Abelian varieties in terms of their associated symplectic data. For simple Abelian varieties over an algebraically closed field of characteristic zero we prove that the natural correspondence introduced by Orlov, which maps equivalences to symplectic isomorphisms, is surjective.