Special Session 2: Abelian Varieties

Organizers: Herbert Lange (FAU, Germany) and Anita Roja (Universidad de Chile, Chile).

Hyperelliptic curves on abelian surfaces
Pawel Borowka

Jagiellonian University, Kraków, Poland

Monday 22, 09:25-10:00

Let A be a general abelian surface and let C be a smooth complex hyperelliptic curve embedded in A. A simple computation shows that the only possibilities for this to occur is when the genus $g(C) = 2, 3, 4, 5$ and A is of type $(1, g - 1)$. The cases $g = 2$ and $g = 3$ are classical. They correspond to the embedding of a genus 2 curve in its Jacobian and the construction of an étale double covering of a genus 2 curve. In the talk we focus on the two remaing cases. We show that indeed for general $(1, 3)$ and $(1, 4)$ polarised surfaces one can find unique hyperelliptic curves up to translation. We give some geometric description of the curves. The $(1, 3)$ case is a joint work with G. Sankaran and the $(1, 4)$ case is a joint work with A. Ortega.

Gauss map, singularities of the theta divisor and trisecants
Giulio Codogni

École Polytechnique Fédérale de Lausanne, Switzerland

Tuesday 23, 09:25-10:00

The Gauss map is a finite rational dominant map naturally defined on the theta divisor of an irreducible principally polarised abelian varieties.
In the first part of this talk, we study the degree of the Gauss map of the theta divisor of principally polarised complex abelian varieties. Thanks to this analysis, we obtain a bound on the multiplicity of the theta divisor along irreducible components of its singular locus. We spell out this bound in several examples,

and we use it to understand the local structure of isolated singular points. We further define a stratification of the moduli space of ppavs by the degree of the Gauss map. In dimension four, we show that this stratification gives a weak solution of the Schottky problem, and we conjecture that this is true in any dimension. This is a joint work with S. Grushevsky and E. Sernesi.

In the second part of this talk, we will study the relation between the Gauss map and trisecant of the Kummer variety. Fay's trisecant formula shows that the Kummer variety of the Jacobian of a smooth projective curve has a four dimensional family of trisecant lines. We study when these lines intersect the theta divisor of the Jacobian, and prove that the Gauss map of the theta divisor is constant on these points of intersection, when defined. We investigate the relation between the Gauss map and multisecant planes of the Kummer variety as well. This is a joint work with R. Auffarth and R. Salvati Manni.

On the signed Euler characteristic property for subvarieties of abelian varieties

Eva Elduque

University of Wisconsin-Madison, USA

Tuesday 23, 08:45-09:20

Franecki and Kapranov proved that the Euler characteristic of a perverse sheaf on a semi-abelian variety is non-negative. This result has several purely topological consequences regarding the sign of the (topological and intersec- tion homology) Euler characteristic of a subvariety of an abelian variety, and it is natural to attempt to justify them by more elementary methods. In this talk, well explore the geometric tools used recently in the proof of the signed Euler characteristic property. Joint work with Christian Geske and Laurentiu Maxim.

On Shimura subvarieties of A_g contained in the Prym locus

Paola Frediani

Università di Pavia, Italy

Monday 22, 10:50-11:25

I will present some results obtained in collaboration with E. Colombo, A. Ghigi and M. Penegini on Shimura subvarieties of A_g generically contained in the Prym locus.

I will explain the construction of $1-$dimensional families of double covers compatible with a fixed group action on the base curve C such that the quotient of C by the group is the projective line. I will give a simple criterion for the image of these families under the Prym map to be a Shimura curve. I will show that this criterion allows us to construct several examples of Shimura curves

generically contained in the Prym locus in A_g for $g < 13$.

Varieties with symmetries and the Discrete Fourier Transform matrix

Santiago López de Medrano

Instituto de Matemáticas, UNAM, Mexico

Monday 22, 08:45-09:20

In the study of a certain type of intersections of real quadrics with dihedral symmetry a relation was stablished between their smoothness and the minors of the Vandermonde matrix with the n-th roots of unity as entries, also known as the Discrete Fourier Transform (DFT) matrix.

In this talk we consider the analogous situation of complex projective varieties with cyclic or dihedral symmetry, for which their smoothness is precisely equivalent to the fact that certain minors of the DFT matrix do not vanish. This question turned out to be very difficult to answer in general and we have obtained only some partial results.

From numerical evidence we arrived to some conjectures and results about those minors. It turned out that our conjectures were true, one of them by and old theorem by Chebotaryov stating that in the case n is prime all the minors are non-zero. The other one we proved ourselves. Recently we have obtained some results for the case where n is a prime power.

This question is of interest in the field of Signal Recognition, starting from work by Terry Tao who rediscovered and applied Chebotaryov's theorem. This was followed by papers in Electrical Engineering journals, among them some recent ones that contain part of our results.

An important part of this research is joint work with Matthias Franz.

Generic Torelli for the Prym map of ramified coverings

Juan Carlos Naranjo

University of Barcelona, Spain

Monday 22, 10:10-10:45

In this talk we will report on a work in collaboration with Angela Ortega. We consider the two remaining open cases on the behaviour of the Prym map of ramified coverings. Namely, let

$$P_{r,g} : \mathcal{R}_{g,r} \longrightarrow \mathcal{A}^{\delta}_{g-1+\frac{r}{2}}$$

be the map sending an irreducible double covering $\pi : D \longrightarrow C$ of a curve C of genus g with $r > 0$ ramification points to the abelian variety $P(D,C) := Ker :$ $JD \longrightarrow JC$, called the Prym variety of the (ramified) covering. The generic

Torelli theorem states that $P_{r,g}$ is generically injective when the dimension of the space of coverings is less or equal to the dimension of the space of polarized abelian varieties. In a fundamental paper Marcucci and Pirola proved this theorem except for the bielliptic case (solved later by Marcucci and Naranjo), the case $r = 4, g = 3$ when the degree is 3 (already considered in the literature) and two isolated cases: $P_{5,2}$ and $P_{2,6}$. We will present the proof of the generic Torelli theorem in these two last situations. In the first we use the base locus of the linear system attached to the theta divisor in combination with some techniques of vector bundles on curves. Instead, in the second case we relate our map with the étale Prym map and we study the fibre along the locus of the intermediate Jacobians of cubic threefolds.

Completely decomposable Jacobian varieties
Jennifer Paulhus

Grinnell College, Iowa, USA

Tuesday 23, 10:10-10:45

Jacobian varieties which can be factored into the product of elliptic curves have interesting applications to rank and torsion questions. Given a curve X with automorphism group G, idempotent relations in the group ring $\mathbb{Q}[G]$ lead to decompositions of the Jacobian of X. In this talk we discuss some recent results obtained from these techniques. Particularly, new computational advances and the study of intermediate covers allow us to determine these decompositions for curves in high genus, and we use that to find many new examples of completely decomposable Jacobians, including families of such curves.

On the connectedness of the singular locus of the moduli space of principally polarized abelian varieties
Sebastián Reyes-Carocca

Universidad de La Frontera, Chile

Tuesday 23, 10:50-11:25

Let $\mathcal{A}_g$ denote the moduli space of principally polarized abelian varieties of dimension $g \geq 3$. In this talk we shall discuss the connectedness of the singular sublocus of $\mathcal{A}_g$ consisting of those abelian varieties which possess an involution different from $-id$. This is a joint work with Rubí E. Rodríguez.