Special Session 1: Algebraic Surfaces

Organizers: Margarida Mendes Lopes (Universidade de Lisboa, Portugal) and Francisco Monserrat (Universidad Politécnica de Valencia, Spain).

Conditions for a complete ideal to be a multiplier ideal in a rational surface singularity

Maria Alberich-Carramiñana

Universitat Politècnica de Catalunya, Spain

Wednesday 24, 08:45-09:20

The multiplier ideals of an ideal $\mathfrak{a} \subseteq \mathcal{O}_X$ are ideals $\mathcal{J}\left(X, \mathfrak{a}^c\right) = \pi_* \mathcal{O}_{X'}\left(\lceil K_\pi - cF \rceil\right)$ where c varies in $\mathbb{R}_{\geq 0}$. Here, K_π is the canonical divisor associated to a log-resolution $\pi : X' \to X$ of the pair $(X, \mathfrak{a})$, and F is the divisor with simple normal crossings support such that $\mathfrak{a} \cdot \mathcal{O}_{X'} = \mathcal{O}_{X'}(-F)$. In particular, they are all complete and contained in $\mathcal{J}(X, \mathcal{O}_X) = \mathcal{J}\left(X, \mathfrak{a}^0\right) = \pi_* \mathcal{O}_{X'}\left(\lceil K_\pi \rceil\right)$ (which is independent of $\mathfrak{a}$).
The purpose of this talk is to address the question, raised in [2], which asks if every integrally closed (complete) ideal which is contained in $\mathcal{J}(X, \mathcal{O}_X)$ is a multiplier ideal, when X is a complex surface which has a rational singularity. Hence a natural question is whether these conditions are also sufficient for an ideal to be a multiplier ideal, or more explicitly, whether any complete ideal $\mathfrak{a} \subset \mathcal{J}(X, \mathcal{O}_X)$ is of the form $\mathfrak{a} = \mathcal{J}\left(X, \mathfrak{b}^c\right)$ for suitable choices of c and $\mathfrak{b}$. On the one hand, this is true in the smooth case (Favre and Jonsson [1] ; Lipman and Watanabe [3]) and in the log-terminal one (Tucker [4]). Notice that log terminal singularities satisfy $\mathcal{J}(X, \mathcal{O}_X) = \mathcal{O}_X$ and are necessarily rational. On the other hand, Lazarsfeld and others proved a necessary condition for this to hold, and they constructed complete ideals on (highly) non-rational singularities (cones over smooth curves of positive genus) which are not multiplier ideals. In the present talk we will answer this question for rational surface singularities.

Bibliography

[1] C. Favre and M. Jonsson, Valuations and multiplier ideals. J. Amer. Math. Soc. 18 (2005), 655–684.

[2] R. Lazarsfeld, K. Lee, and K. E. Smith: Syzygies of multiplier ideals on singular varieties. Michigan Math. J. 57 (2008) 511-521. Special volume in honor of Melvin Hochster.
[3] J. Lipman and K.-i. Watanabe: Integrally closed ideals in two-dimensional regular local rings are multiplier ideals. Math. Res. Lett., 10 (2003), 423-434.
[4] K. Tucker: Integrally closed ideals on log terminal surfaces are multiplier

Coverings of rational ruled normal surfaces

Enrique Artal Bartolo

Universidad de Zaragoza, Spain

Monday 22, 15:25-16:00

In this talk, we treat the use of arithmetic, geometric, and combinatorial techniques to compute the cohomology of Weil divisors on a special class of normal surfaces, the so-called toric ruled surfaces. We study the behavior of Picard group under weighted blow-ups and we make use of generalized Riemann-Roch formulas, combined with combinatorial computations for weighted homogeneous polynomials. These computations are used to study the topology of cyclic coverings of such surfaces ramified along Q-normal crossing divisors. It is a joint work with J.I. Cogolludo and J. Martín-Morales.

New techniques on linear series on irregular surfaces

Miguel Ángel Barja

Universitat Politècnica de Catalunya, Spain

Tuesday 23, 16:50-17:25

I will explain new techniques for the study of linear series on surfaces of maximal Albanese dimension and how we can use them to give new geographical Severi-type inequalities and classify the limit cases. These are the initial step for an induction process to extend them to higher dimensions. This is a joint work with Lidia Stoppino and Rita Pardini.

Semistable fibrations over an elliptic curve with only one singular fibre

Abel Castorena

UNAM, Mexico

Tuesday 23, 15:25-16:00

In this talk we describe a construction which give rise to the existence of semistable fibrations over an elliptic curve with a unique singular fibre. For this construction we use the monodromy of certain ramified (non Galois) covers $C \to E$, where E is an elliptic curve.

Super-rigid Affine Fano Varieties

Jihun Park

IBS / POSTECH, South Korea

Wednesday 24, 09:25-10:00

Let X be a projective normal $\mathbb{Q}$-factorial variety of Picard number 1 and S be a prime divisor on X. The affine variety $X \setminus S$ is called an affine Fano variety if the pair (X, S) has purely log terminal singularities and $-(K_X + S)$ is ample. Furthermore, the affine Fano variety $X \setminus S$ is said to be super-rigid if the following two conditions hold.

- For every affine Fano variety $X' \setminus S'$ with completion X' and boundary S', if there exists an isomorphism $\phi\colon X \setminus S \cong X' \setminus S'$, then ϕ is induced by an isomorphism $X \cong X'$ that maps S onto S'. In particular, one has $\mathrm{Aut}(X \setminus S) = \mathrm{Aut}(X, S)$.
- The affine Fano variety $X \setminus S$ does not contain relative affine Fano varieties over varieties of positive dimension.

In this talk, examples and non-examples of super-rigid affine Fano varieties are demonstrated. In particular, we will mainly consider del Pezzo surfaces in 3-dimensional weighted projective spaces. These examples lead us to a folklore conjecture that every automorphism of the complement of a smooth cubic surface in $\mathbb{P}^3$ comes from the automorphism group of $\mathbb{P}^3$.
This is a joint work with Ivan Cheltsov and Adrien Dubouloz.

A surface with $p_g = q = 2$ and $K^2 = 8$ which is not uniformized by the bidisk

Carlos Rito

Universidade do Porto, Portugal

Tuesday 23, 14:45-15:20

Complex algebraic surfaces of general type with the lowest possible value of the holomorphic Euler characteristic $\chi = 1$ are far from being classified. For these surfaces the geometric genus p_g equals the irregularity q, and the Bogomolov-Miyaoka-Yau inequality implies $K^2 \leq 9$, where K is a canonical divisor. The ones in the line $K^2 = 9$ are known to be quotients of the complex unit ball by a lattice in $PU(2, 1)$.
All the examples of surfaces with $\chi = 1$ and $K^2 = 8$ known so far are uniformized by the bidisk $\mathbb{H} \times \mathbb{H}$, where $\mathbb{H}$ is the Poincaré upper half-plane.
In this talk I will explain the construction of a surface with $p_g = q = 2$ and $K^2 = 8$ which is not uniformized by the bidisk.
This is joint work with F. Polizzi and X. Roulleau.

Elliptic fibrations on K3 surfaces and linear systems of curves on rational surfaces

Cecília Salgado

Universidade Federal do Rio de Janeiro, Brazil

Tuesday 23, 16:10-16:45

Let X_0 be a rational elliptic surface and X a K3 surface given by a double cover of it. Then X admits a non-symplectic involution ι, namely the double cover involution. Moreover, X admits at least one elliptic fibration, which is induced by the fibration on X_0. We analyze the other elliptic fibrations on X by studying linear systems of curves on X_0. We will discuss how the effect of ι on the elliptic fibrations influences the possible outcomes for linear systems of curves on the rational elliptic surface. (This is joint work with A. Garbagnati).

Complex Ball quotients and hyperkähler fourfolds

Alessandra Sarti

Université de Poitiers, France

Monday 22, 14:45-15:20

In a famous paper of 2011 Allcock, Carlson and Toledo describe the moduli space of smooth cubic threefolds as a 10-dimensional ball quotient. We show how the 10-dimensional ball quotient can also be described as the moduli space of certain hyperkähler fourfolds with a non-symplectic automorphism of order three. We then completely describe the hyperkähler fourfolds and we identify them with the Fano variety of lines of cubic fourfolds that are triple covers of the 4 dimensional complex projective space ramified on a smooth cubic threefold. We finally describe degenerations of the hyperkähler fourfolds which are related to degenerations of the smooth cubic threefolds to the nodal and the chordal locus in the complex ball quotient. This is a joint work in progress with S. Boissière and C. Camere.

p_g Vector Bundles on Normal Surfaces

Kevin Tucker

University of Illinois, Chicago, USA

Monday 22, 16:55-17:30

It is well-known that complete (or integrally closed) ideals on a rational surface singularities satisfy a number of special properties – for example, a product of complete ideals is complete. While similar statements fail for general normal surfaces, they hold for a special class of complete ideals introduced by Okuma-Watanabe-Yoshida called p_g ideals. In this talk, I will discuss a higher-rank generalization of these concepts, called p_g vector bundles and some of their basic properties. This is joint work with Lawrence Ein.

Optimal bounds for T-singularities in stable surfaces

Giancarlo Urzúa

Pontificia Universidad Católica de Chile, Chile

Monday 22, 16:10-16:45

Kollár and Shepherd-Barron (1988) introduced a natural compactification to the Gieseker moduli space of surfaces of general type, which is analogous to the Deligne-Mumford (1969) compactification of the moduli space of curves of genus g¿1. This compactification is coarsely represented by a projective scheme (due to Kollár 1990) because of Alexeev's proof of boundedness (1994). Thus we have a proper KSBA moduli space of stable surfaces, which includes classical canonical surfaces of general type. In particular, after fixing the self-intersection of the canonical class, we have a finite list of singularities appearing on stable surfaces. It is hard to write down that list.
T-singularities are cyclic quotient singularities of the form $1/dn^2(1, dna-1)$(with $\gcd(n, a) = 1, n > 1$). This is a remarkable set of singularities in stable surfaces, since they are the key singularities showing up in normal degenerations of surfaces in the KSBA compactification. When $d = 1$, they are also called Wahl singularities (i.e. cyclic quotient singularities having a smoothing with Milnor number equal to zero).
In a jointly work with Julie Rana, we explicitly bound T-singularities on normal projective stable surfaces W with one singularity. This bound depends on K^2, and it is optimal when W is not rational. We classify and realize surfaces attaining the bound for each Kodaira dimension of the minimal resolution of W. This talk will be about that work.